Modality: Patterns, Theories, Logic I

Diti Bhadra
University of Minnesota
bhadra@umn.edu
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Material partially based on Portner (2009), Chapters 1 & 2.

1 Introduction

- *Modality* allows us the capacity to talk about objects and events that are displaced in space and time, as well as in actuality and potentiality.

- Given one of the design features of language is *displacement* (Hockett 1960), modal expressions are all pervasive in grammars.

- Modal auxiliaries and verbs: *can, could, should, must, would, need to, have to*
  Modal adverbs: *possibly, necessarily, maybe, probably, certainly*
  Modal adjectives, nouns: *possible, necessary, probable, certain, need, necessity, possibility*
  Propositional attitude verbs: *think, believe, hope, know, pleased*
  Generics, habituals, individual-level predicates: *A linguist studies languages, Diti teaches semantics, Ayesha is hilarious*
  Conditionals: *If Utpal writes a book on NPIs, then...*
  Tense and aspect: *I will go to the store, We are listening to a talk, We have taken a course on modality*
  Evidentiality: *It seems like there's a tiger in that shed*
  Covert modality with infinitives: *Tim knows how to solve the problem* (Bhatt 2008: "Tim knows how he *can* solve the problem.")

- Let's look at some examples from Hindi:

(1) a. *Abhi woh ghar pe **hoga**.* $\qquad\qquad$ *hoga*$_{epistemic}$
         'Now he must be home.'

   b. *Abhi usse ghar pe hona **chahiye**.* $\qquad\qquad$ *chahiye*$_{deontic/epistemic}$
         'He should be home now.'

   c. *Woh 10 minut mein 2 km daur **sakta** hain.* $\qquad\qquad$ *sakta*$_{ability/deontic}$
         'He is able to/he may run 2kms in 10 minutes.'

d. *Mujhe 10 minut mein 2 km daur**na hai**.*

V-INF $hain_{(covert)deontic/bouletic/teleological}$

'I am required to/want to/have to run 2kms in 10 minutes.'

e. *Tum kehte toh woh kar **leta**.*   V-$ta_{epistemic/counterfactual}$

'If you would have said, he would have done it.'

f. *Tumhe yeh kaam karna **padhega**.*   V-INF $padhega_{deontic/teleological}$

'You have to/need to do this work.'

(2) *Ram yeh kar **sakta** hain.*

a. Ram can do it, i.e. he is physically able to.

b. Ram can do it, i.e. now he is allowed to/I am giving him permission to.

c. Ram may/might do it, i.e. I am not sure if he certainly will but there is a possibility given what I know about him.

d. Ram may do it, i.e. the moral ethics we live by do not prevent him from doing this.

e. Ram can do this to achieve a particular goal.

✦ What we are already noticing is massive amounts of ambiguity/underspecification in *modal flavor*.

    ☞ Then should we consider the existence of these patterns as a result of accidental polysemy?

    ☞ Kratzer (1981, 1991): No! This is the result of **contextual dependency**.

    ☞ Modals by themselves have a skeletal meaning denotation; together with essential components from the context, a modal gets the particular flavor that it has in that particular context.
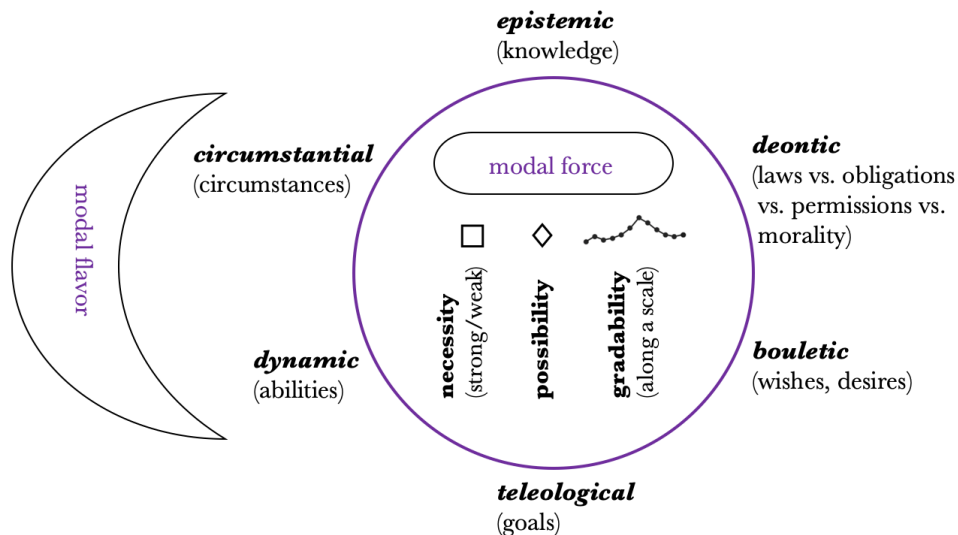

Figure 1: Modal *force* vs. modal *flavor*

- Any theory of modality in natural language has to explain how to account for these different flavors within the same framework, and correlate them with differences in modal force.

- We are going to look at two influential schools of thought bearing on this issue: modal logic (accessibility relations in particular), and natural language semantics (possible worlds in particular).

- There are many many aspects and facets of modality we will not be able to cover in this workshop: their interaction with scopal operators like *negation, quantifiers, conditionals, questions*, etc, their direct interaction with tense and aspect, their interaction with disjunction (*free choice*), clause-type related phenomena like *modal subordination*, relationship with *imperatives*, the very related phenomena of *evidentiality*, etc.

2   Accessibility Relations

- *Logic* comprises studying systems of reasoning; *Modal Logic* focuses on reasoning involving the concepts of necessity, possibility, implication, obligation, etc. (Portner 2009).

- Modal logic is not the same thing as the linguistics of modal expressions!

  - Modal logic aims to capture patterns of reasoning irrespective of natural language words like *must, possible, sakna, hona, zaroori, dorkar, uchit, uchitam*, etc.

  - Once you start studying the formal properties of operators like $\Box$ and $\Diamond$, it can take you far far beyond the semantics of human language.

- In a modal logic language (MLL; Portner 2009), we first begin with some essential tools:

  (3)   Included in such a MLL are:
  - a.   Infinite number of propositional variables: $p, q, r, s, t \dots$
  - b.   Negation: if α is a sentence in MLL, then so is $\neg\alpha$.
  - c.   Conjunction, disjunction, conditionals: if α and β are sentences of MLL, then so are $\alpha \wedge \beta$, $\alpha \vee \beta$, $\alpha \rightarrow \beta$.
  - d.   Necessity and Possibility: if α is a sentence of MLL, then so are $\Box\alpha$ and $\Diamond\alpha$.

- Note that some of these are *derivable* from other, more basic operators:

  (4)   a. $\alpha \wedge \beta = \neg(\neg\alpha \vee \neg\beta$
  - b. $\alpha \rightarrow \beta = (\neg\alpha \vee \beta)$
  - c. $\Box\beta = \neg\Diamond\neg\beta$

- Next, we come to two concepts that will get the "modal" part of things rolling (Hughes and Cresswell 1996):

(5)   FRAMES

       A frame F is a pair $< W, R >$ where W is (usually) a set of possible worlds, and R is a relation on W.

✦ This relation R is ultra important because it provides the mechanism for the worlds to "talk" to each other. For example:

(6)   a.   **Reflexive frame**: $< W, R >$ is a reflexive frame iff for every $w \in W$, R(w, w).

      b.   **Symmetrical frame**: $< W, R >$ is a symmetrical frame iff for every w and $u \in W$, if R(w, u), then R(u, w).

      c.   **Serial frame**: $< W, R >$ is a serial frame iff for every $w \in W$, there is a $u \in W$ such that R(w, u).

      d.   **Transitive frame**: $< W, R >$ is a transitive frame iff for every $w, u, v \in W$, if R(w, u), and R(u, v), then R(w, v).

      e.   **Equivalence frame**: $< W, R >$ is an equivalence frame iff it is a reflexive, symmetrical, and transitive frame.

✦ Think of these frames as ways a world can talk to/access its friends that are far away from it.

     ☞ Different types of R thus gives us different types of ***accessibility relations*** (cf. Kripke semantics – a system created by Saul Kripke and Andre Joyal in late 1950s).
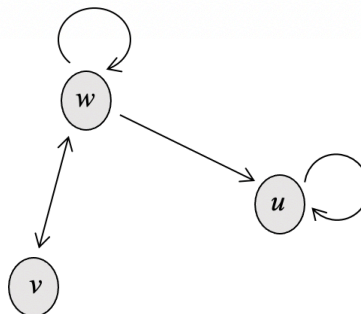

Figure 2: Accessibility between worlds

✦ Can we figure out the different relations/frames that exist between the worlds *w, u, v*?

✦ With the concept of a FRAME in place, we can define a MODEL:

(7)   MODEL

       A model M is a pair $< F, V >$, where F is a frame and V is a valuation function that takes a proposition in a $w \in W$ and returns a value of 1 or 0.

❖ For example, $V(w, p) = 1$, $V(u, q) = 0$, etc.

❖ We can do this for all atomic propositions as well all derived complex propositions containing operators like $\wedge, \vee, \rightarrow, \Box, \Diamond$.

❖ We also have the useful logical property of ***validity*** in modal logic:

(8)   a.   A sentence α is VALID in a MODEL $M = <<W, R>, V>$ iff $[\![\alpha]\!]^{w,M} = 1$ for $\forall w \in W$.

        b.   A sentence α is VALID on a FRAME F iff, for every valuation function V, α is valid in the model $M = <F, V>$.

❖ Thus, validity in a model is separate from validity on a frame (changing the valuation function changes the latter, for instance).[1]

❖ Take a reflexive frame, and if a sentence is valid on such a frame then it is T-VALID:

(9)   A T-VALID sentence: $\Box p \rightarrow p$
Natural language counterparts:
    a.   If Ram must be at home now, then he is at home now.
        (where $must_{epistemic}$)

    b.   If Sita must eat 10 rasgullas to win, then she will eat 10 rasgullas.
        (where $must_{deontic}$)

❖ Only (9-a) is T-VALID because the sentence is true, while (9-b) is not T-VALID because the sentence is not true.

    ☞ This tells us that T-validity can be useful for epistemic logic, but not for deontic logic.

❖ Take a serial frame, and if a sentence is valid on such a frame then it is D-VALID:

(10)   A T-VALID and D-VALID sentence: $\Box p \rightarrow \Diamond p$
Natural language counterparts:
    a.   If Ram must be at home now, then he might be at home now.
        (where $must_{epistemic}, might_{epistemic}$)

    b.   If Sita must eat 10 rasgullas to win, then she may eat 10 rasgullas.
        (where $must_{deontic-obligation}, may_{deontic-permission}$)

❖ With the flavors and subflavors as indicated, the sentence with deontic modals in (10-b) being D-VALID suggests that D-validity can be a property of deontic logic.

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[1]There can be different kinds of validity based on the specific types of frame: K-VALID, T-VALID, B-VALID, D-VALID, S4-VALID, S5-VALID (Lewis 1918, Gödel 1933, Blackburn et al. 2002).

↬ Note that the sentence with the epistemic modals are thus both T-valid and D-valid (more on this on page 6 below).

❧ Our main stars for today, $\Box$ and $\Diamond$, impose a **layer of quantification** on the accessibility relations between worlds:

(11) $[\![\Box p]\!]^{w,M} = 1$ iff for **every world** in W accessible to w, p holds (/is true) in all of those worlds.

↬ Crucially note that we have not said *what type* of accessibility relation holds between the worlds – this could be any type!

↬ So what is the $\Box$ giving us – just the *quantity* of accessible worlds where the proposition in its scope holds.
This is thus a natural way to think about **modal force**.

(12) $[\![\Diamond p]\!]^{w,M} = 1$ iff for **some world** in W accessible to w, p holds (/is true) in that world.

❧ Now it's time to wonder: where does **modal flavor** then come from in this framework of modal logic?
☞ From the refined definitions of the accessibility relation R in terms of knowledge, rules, permissions, obligations, goals, desires, circumstances, abilities, etc.

❧ Thus, our frames and accessibility relations can now have special designations: (Note: I am using Portner's style of descriptions below (cf. Portner 2009) but most styles are sons and daughters of the formalizations in Kripke semantics; cf. also Hughes and Cresswell 1996, Blackburn et al. 2002).

(13)   **Epistemic frame**
        $F = <W, R>$ is an epistemic frame iff for some individual i:
        a. W = the set of possible worlds conceivable by humans.[2]
        b. R = the relation which holds between two worlds w and u iff everthing which i knows in w is also true in u.

❧ This R inside this EPISTEMIC FRAME can be called an EPISTEMIC ACCESSIBILITY RELATION.

❧ What about the different properties of frames we learnt in (6)?
☞ They can be applied to each of these relations.

❧ For example, *knowledge* is considered to have the property of reflexivity, because if an agent *knows* a proposition in w, then it is true/is a fact in w.
– thus, we can say that the epistemic frame is reflexive.

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[2]Leibniz' work on modal metaphysics takes a possible world to a complete way that the universe could be throught its history; possible worlds accessible from our world could have minimal details changed (like maybe today is a Tuesday) or be completely different (maybe dinosaurs are still alive). Linguists ignore inconceivable worlds.

✦ Actually, if we assume an idealized account of knowledge where agents are perfect reasoners with infinite memory, then we can say that the epistemic frame actually contains an *equivalence* relation.
  – (for example, recall that sentences with epistemic modals were both T-VALID and D-VALID to start with. cf. (10))
  – but we know that that knowledge derived from different types of evidence and inferences sometimes admit errors, etc. (von Fintel and Gillies, 2010; Mihoc, Bhadra, and Falaus, 2019)

✦ With similar tools, we can define a DEONTIC FRAME and a corresponding DEONTIC ACCESSIBILITY RELATION inside it (assuming we are in the realm of rules for instance):

  (14) **Deontic frame**
        $F = < W, R >$ is an deontic frame iff for some system of rules r:
        a. $W =$ the set of possible worlds conceivable by humans.
        b. $R =$ the relation which holds between two worlds w and u iff all of the rules which are established by r (the relation that associates each world in W with a set of rules) in w are followed in u.

✦ Again, thinking about properties of this frame, deontic frames are *serial*:
  – by invoking seriality we obliterate the possibility of having an inconsistent set of rules
  – because for every rule, there is a world in which the rule is followed (because the property of seriality requires that there be a corresponding world for every world accessible by the deontic accessibility relation).

✦ A disclaimer again: this is an idealized notion of a deontic relation, because we might find ourselves with contradictory requirements in reality.

✦ Now, with these tools in place, how do we apply them to our modal expressions?

  (15) a. Necessity modals ($\Box$): *must, should, would, zaroori*, V-INF *chahiye*, V-INF *padhega, nischoi, dorkar, uchit, uchitam*, etc.
        b. Possibility modals ($\Diamond$): *may, might, can, could, sakna, shayad, hoyto, bodhoy*, etc.

✦ The central insight: for each modal flavor and *subtypes* of each flavor, there has to be a separate accessibility relation.

✦ Formally:

  (16) $R_{epis}(w) = \{w' | w'$ is a world in which all the known facts in w hold$\}$

  (17) $R_{deontic}(w) = \{w' | w'$ is a world in which all the rules in w are followed$\}$

✦ Within deontic necessity, there is a distinction between **strong** and **weak** necessity (*must, necessary, have to*, etc. vs. *should, ought*).

- Test 1 for detecting strength: strong necessity modals can reinforce weak ones but not vice-versa (von Fintel and Iatridou 2008):

  (18)　a.　You should wash your hands, in fact you must.

  　　　b. ??You must wash your hands, in fact you should.

- Test 2 for detecting strength: Weak necessity modals are compatible with the negation of strong ones, but not vice versa (von Fintel and Iatridou 2008):

  (19)　a.　You ought to/should do the dishes, but you don't need to/have to.

  　　　b. ??You need to/have to do the dishes, but it's not the case that you should/ought to.

- For an analysis of differential strength in deontic necessity in Bangla (with connections to Telugu and Hindi), see Bhadra and Banerjee (2020).

- Within modal logic, can we capture this relationship?: $must\ p \models should\ p$

- Let r_{must} be the set of rules on which the accessibility relation of $must$ – $R_{must}(w)$ is based on; and let r_{should} be the set of rules on which the accessibility relation of $should$ – $R_{should}(w)$ is based:

  (20)　$R_{must}\ (w) = \{w' | \forall w'\ s.t.\ R(w, w'), V(p)(w') = 1\}$ 　　　　　$\equiv \bigcap r_{must}$

 (21)　$R_{should}\ (w) = \{w' | \forall w'\ s.t.\ R(w, w'), V(p)(w') = 1\}$ 　　　　　$\equiv \bigcap r_{should}$

- Both are deontic necessity modals ($\square$), thus crucially there is no difference in the denotations of the modals;
  – the main action (/difference) is in the sets of rules behind the accessibility relations.
  (Keep this insight in mind – it will carry over to the other school of thought we will be exploring tomorrow).

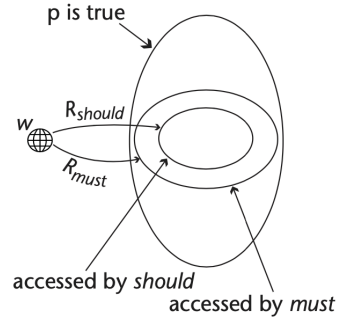- Steps (let's name r_{must} as M and r_{should} as S):

  (22)　a.　if $M \subseteq S$

  　　　b.　then $R_{should}(w) \subseteq R_{must}(w)$

  　　　c.　i.e. $\bigcap S \subseteq \bigcap M$

  　　　d.　i.e. the subset relation reverses once we take intersections

- Now look at Portner (2009)'s representations of the relationship:

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∩M= ● ∩ ○
(dark-or-light gray)

∩S= ●
(dark gray)
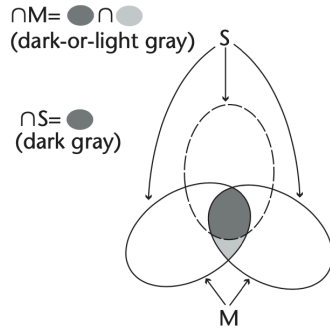
Intersection reverses the subset relation

p is true

R_{should}

w

R_{must}

accessed by *should*

accessed by *must*

Deontic *must p* entails deontic *should p*

- ➤ In the first figure: M contains less rules thus only the two solid ovals, while S contains more rules thus all three ovals.

- ➤ Thus, $\bigcap M$ is larger than the $\bigcap S$, confirming the subset relationship in (22-c).

- ➤ **Insight:** the more things you care about, the smaller the intersection of worlds will be where all of those things are true.

- ➤ Thus, we can confirm *must p* $\models$ *should p*.
  – provided that there are at least two different kinds of deontic accessibility relations
  – figure on the right

- ➤ But deontic modality can have many other subtypes, and thus even these two deontic accessibility relations are not enough!
  – apart from the two kinds within $R_{deontic-rules}$, there has to be $R_{deontic-permissions}$, $R_{deontic-obligations}$, $R_{deontic-morals}$, etc.

  (23) Just splitting apart deontic *must*:
      a. [In view of the laws of Minnesota], drivers must yield to pedestrians.
      b. [In view of the traditions of our family], you, as the youngest child, must touch the feet of all elders.
      c. [In view of the rules of student-teacher relationships], you must not yell at your teachers.

- ➤ Kratzer (1977) points out that the kind of restrictions in [ ]-s that determine differences in accessibility relations can be infinitely many.

- ➤ **So what is starting to look like a problem here?**
  ☞ Masssive over-generation!

- ➤ Additionally, not to forget that there are several other flavors outside of deontic modality!:

  (24) a. Dogs **can** swim. (ability/dynamic)

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b. Given how much you love semantics, you **should** attend the modality workshop. (<u>desire</u>)

c. To get into JNU, you **have to** study hard. (<u>goal</u>)

d. A pandemic **may** eventually wipe out large sections of humanity. (<u>history</u>)

(25) $R_{bouletic}(w) = \{w' | w'$ is a world in which all the desires of an agent i in w are satisfied$\}$

✦ The massive ambiguity/underspecification problem that we started out with is exacerbated by the possibility of the generation of multiple accessibility relations for the same modal:

(26) *Ram yeh kar* **sakta** *hain.*

a.	$R(w, v)$, where $R_{dynamic}$	$\{w_1, w_2, w_3, w_4, \ldots\}$
b.	$R(w, v)$, where $R_{deontic-permissions}$	$\{w_5, w_6, w_7, w_8, \ldots\}$
c.	$R(w, v)$, where $R_{epistemic}$	$\{w_9, w_{10}, w_{11}, w_{12}, \ldots\}$
d.	$R(w, v)$, where $R_{deontic-morality}$	$\{w_{13}, w_{14}, w_{15}, w_{16}, \ldots\}$
e.	$R(w, v)$, where $R_{teleological}$	$\{w_{17}, w_{18}, w_{19}, w_{20}, \ldots\}$

✦ And then within modal logic, what prevents 10 other accessibility relations from being generated for *sakna*?[3]

✦ Thus, we understand the nature of the formal relations and properties better but have we made real progress towards providing an account of **modality in natural language**?

✦ 👩: Unfortunately, no! Because our current logical system predicts many many more meanings for modals than they actually have.

✦ Enter: the field-changing work of Angelika Kratzer.
(tomorrow's journey)

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[3]We have not even discussed the *indexicality* of modals (relativization to contextual parameters like speakers, hearers, times, locations); imagine how much more over-generation there can be once these factors are wired into the semantics.

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Modality: Patterns, Theories, Logic II

Diti Bhadra
University of Minnesota
bhadra@umn.edu
Semantics Workshop at JNU, 09/2020

Material partially based on Portner (2009), Chapter 3.

Yesterday, we talked briefly about the interaction of modality with indexicality. For people interested in following up, there are 'judge-based' semantics inspired by indexicality in various forms (Lewis 1979, Chierchia et al. 1989, Lasersohn 2005, Stephenson 2007, MacFarlane 2011, Bhadra 2017, 2020).

I also mentioned that there are probability-theory-based accounts of modality. For people interested in this, more of this approach exists in the epistemic domain (Frank 1996, Swanson 2006, Yalcin 2007) than in the general modal domain (Lassiter 2011, 2017).

1   Taking stock

- ✢ <u>Goal</u>: Within any possible worlds-based theory of modality, the foundational issue concerns how to identify the correct set of worlds over which a particular modal expression quantifies.

    - ➥ How did we accomplish this in modal logic?

- ✢ In a framework of *relative modality*, pragmatics plays a much more significant role:

    (1)   *Tumhe yeh kaam karna **padhega**.*
    ↪ [In view of the requirements that are imposed upon you,] you have to do this work to fulfill those requirements.   (deontic)
    ↪ [In view of the goals you are trying to achieve,] you have to do this work to fulfill those goals.   (teleological)

- ✢ In a modal logical approach, *padhega* would be considered to be an example of *lexical ambiguity*, and the context merely sorts out which meaning the speaker has in mind.

- ✢ Kratzer (1977, 1981) would disagree and say *padhega* is a single word (very skeletal in meaning) lexically, and the context determines whether the accessible worlds are your requirements-worlds or your teleological-worlds.

- ✢ So note that some of the fundamental insights from modal logic are going to be used in this approach as well.

2   Relative Modality/Possible Worlds Semantics

- A modal expression is **doubly relative** to two 'conversational backgrounds', namely:
  (i) a set of accessible worlds *(modal base)*
  (ii) an ordering of those worlds *(ordering sources)*

- How do we derive and compute this conversational backgrounds?

  (2)   *In view of what we know,*
  In view of what is required,
  In view of my desires,
  In view of my goals,
  .
  .

- These kinds of clauses are **functions *f* from worlds to sets of propositions**:

  (3)   For any world *w*, *f(w)* can be the set of propositions a speakers knows in *w*; the set of propositions that are the rules in *w*; set of propositions that describes the speaker's desires in *w*...

  (4)   Some central concepts:
  a. $f(w) = \{p_1, p_2, p_3, p_4, p_5, \ldots\}$
  b.   Then $\bigcap f(w) = p_1 \cap p_2 \cap p_3 \cap p4 \ldots = \{w_{17}, w_{15}, w_{22}, w_{48}, \ldots\}$
  c.   *f* is a conversational background that serves as a parameter of interpretation, while $\bigcap f(w)$ provides us the quantificational domain of a modal.

- We can see an easy link between conversational backgrounds and accessibility relations:

  (5)   For any worlds *u, v*: *u* and *v* are accessible from the world of evaluation *w* iff every proposition in *f(w)* is true in *u* and *v*.

- Thus:

  (6)   a.   The set of worlds accessible from *w* is $\bigcap f(w)$!
  b.   This $\bigcap f(w)$ is called the ***modal base***.

- Now let's see how this set constituting the modal base is used as the quantificational domain for a modal statement:

  (7)   a. $⟦\text{you this work do-INF \textbf{padhega}}⟧^{w,c,f} = 1$ iff $\forall v \in \bigcap f(w), ⟦\text{you this work do-INF}⟧^{w,c,f} = 1$
  b. $⟦\text{you this work do \textbf{sak-te}}⟧^{w,c,f} = 1$ iff $\exists v \in \bigcap f(w), ⟦\text{you this work do}⟧^{w,c,f} = 1$

- Note that we are still very much using the essence of the traditions of modal logic.

- For example, to highlight the similarities, we can define an accessibility relation from the conversational background (cf. Portner 2009):

  (8)   a. $R_f(w, w')$ iff $w' \in \bigcap f(w)$.

        b.   Then $\Box \alpha$ and $\Diamond \beta$ come in to do their work as usual.

- We said $\bigcap f(w)$ constitutes the *modal base*, and herein lies the representation of diversity in modal flavors:

  (9)   Kratzer (1977, 1981)'s list of modal bases (but we can make even finer distinctions) and an addition (g):

        a.   If *f(w)* is the set of propositions the speaker knows, $\bigcap f(w)$ is an *epistemic modal base*.

        b.   If *f(w)* is the set of propositions that constitute rules, morals, obligations, etc, $\bigcap f(w)$ is a *deontic modal base*.

        c.   If *f(w)* is the set of propositions that constitute the desires of the speaker, $\bigcap f(w)$ is *a bouletic modal base*.

        d.   If *f(w)* is the set of propositions that constitute the goals of the speaker, $\bigcap f(w)$ is a *teleological modal base*.

        e.   If *f(w)* is the set of propositions constituting the circumstances/reality, $\bigcap f(w)$ is a *circumstantial modal base*.

        f.   If *f(w)* is the set of propositions constituting the expectations about what w is like, $\bigcap f(w)$ is a *stereotypical modal base*.

        g.   If *f(w)* is the set of propositions constituting the speaker's beliefs in w, $\bigcap f(w)$ is a *doxastic modal base*.

- We directly used the quantificational language in (7), but actually Kratzer's formalizations are in terms of *entailment* and *compatibility*:

  (10)   If N is a necessity modal and α is of the form Nβ,
         $[\![\alpha]\!]^{c,f} = \{w : f(w) \text{ entails } [\![\beta]\!]^{c,f}\}$ (proposition talk)
         (i.e., $= \{w : \bigcap f(w) \subseteq [\![\beta]\!]^{c,f}\}$)   (world talk)

  (11)   If P is a possibility modal and α is of the form Pβ,
         $[\![\alpha]\!]^{c,f} = \{w : f(w) \text{ is compatible with } [\![\beta]\!]^{c,f}\}$ (proposition talk)
         (i.e., $= \{w : \bigcap(f(w) \cap [\![\beta]\!]^{c,f}) \neq \varnothing\}$)   (world talk)

- The compatibility restriction means that at least in one world in the speaker's epistemic modal base (let's say), β is true.

- Now you may be wondering if all of those properties we learnt about frames in modal logic are applicable here or not:

3

(12) Properties of *modal bases* (Hughes and Cresswell 1996, Kaufmann et al. 2006, Portner 2009):

    a. **Reflexivity** corresponds to the property of **Realism**: A conversational background f is realistic iff for all $w \in W$, $w \in \bigcap f(w)$.

    b. **Seriality** corresponds to the property of **Consistency**: A conversational background f is consistent iff for all $w \in W$, $\bigcap f(w) \neq \varnothing$.

    c. **Transitivity** corresponds to the property of **Positive introspection**: A conversational background f displays positive introspection iff for all $w, w' \in$, if $w' \in \bigcap f(w)$, then $\bigcap f(w') \subseteq \bigcap f(w)$.

✦ Let's see how the last property works.

✦ Kaufmann et al. (2006): Positive introspection is the requirement that at each world compatible with what the speaker believes (i.e., each world in R_w), she has all the beliefs that she actually has at w (and possibly more).

    ↪ Formally, this means that for each such belief-world w', the speakers doxastic modal base $R_{w'}$ is a **subset** of the actual modal base R_w.

    ↪ Why?!

✦ Let's assume the speaker believes the following propositions in w:
$f(w) = \{$*JNU is in New Delhi* (p_1),
Agra is 40kms from New Delhi (p_2),
My neighbor has been stealing my electricity (p_3) $\}$

✦ R_w will be the corresponding doxastic modal base $(\equiv \bigcap f(w))$, which is a set of worlds $(\{w_{13}, w_{14}, w_{15}\})$ where p_1, p_2, p_3 are true.

✦ Now, look at *positive introspection* again: it demands that in each world in R_w, the speaker has all the original beliefs (the set of p_1, p_2, p_3) and possibly more beliefs.

    ↪ In each of the worlds in $\{w_{13}, w_{14}, w_{15}\}$, the speaker believes p_1, p_2, p_3 and possibly more.

    ↪ Therefore, **Rw_{13}, Rw_{14}, Rw_{15} should all be subsets of R_w!**
(Remember the 'insight' on page 9 of my yesterday's handout: the more propositions you have to make true, the smaller the set of worlds will be from the generalized intersection of those propositions).

✦ Thus, we can also write the definition of positive introspection in (12-c) as follows (Kaufmann et al. 2006):

(13) For all $w' \in R_w$, we have $R_{w'} \subseteq R_w$.

✦ This is beautifully equivalent to the property of transitivity with the accessibility relation:

(14)   If $w\text{R}w'$ and $w'\text{R}w''$, then $w\text{R}w''$.
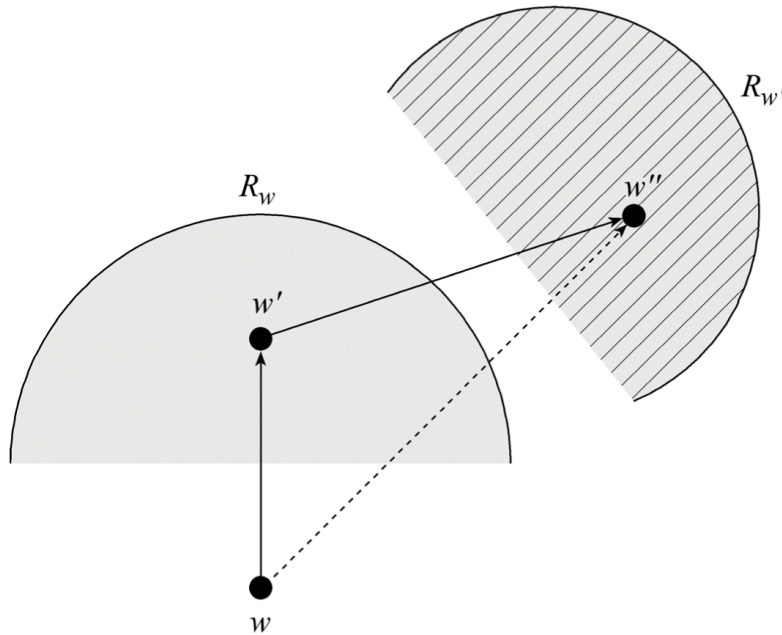
A violation of transitivity. No direct accessibility link leads from w to w'';
$R_{w'}$ is not a subset of R_w.

- The modal bases are indicated as partial spheres (Kaufmann et al. 2006: Figure 1).

- Positive introspection fails here because **$R_{w'}$ is not fully contained in R_w**.

  - There is a world w'' that is inside R'_w but not inside R_w, violating the necessary subset relation in (13).

  - That means the speaker believes p_1 in world w, and believes it in w', but does not believe it w''.

  - i.e. there is no accessibility between w and w''.

  - Since w'' is compatible with what the speaker believes in w, we have a speaker who thinks at w that they do not believe p_1 but actually does.

- To rule out such scenarios, positive introspection is imposed as a **condition** on epistemic and doxastic modal bases.

- There is also the property of **negative introspection** (requirement that there be no world compatible with what the speaker believes at which she holds any beliefs that she does not actually hold) – the reverse of the subset relation in (13) – that is also imposed as a condition on epistemic and doxastic modal bases.

- Thus we have the following kinds of correspondences between properties of modal bases and accessibility relations, and the corresponding axioms that are guaranteed to hold (Kaufmann et al. 2006: Table 1):

5

Modal Base	Accessibility Relation	Axiom	
consistency $R_w \neq \varnothing$	seriality $(\exists w')\, wRw'$	$\Box_R \varphi \to \Diamond_R \varphi$	(D)
realism $w \in R_w$	reflexivity wRw	$\Box_R \varphi \to \varphi$	(T)
total realism $R_w = \{w\}$	identity $wRw' \Leftrightarrow w = w'$	$\Box_R \varphi \leftrightarrow \varphi$	
positive introspection $w' \in R_w \Rightarrow R_{w'} \subseteq R_w$	transitivity $wRw' \wedge w'Rw'' \Rightarrow wRw''$	$\Box_R \varphi \to \Box_R \Box_R \varphi$	(4)
negative introspection $w' \in R_w \Rightarrow R_w \subseteq R_{w'}$	euclidity $wRw' \wedge wRw'' \Rightarrow w'Rw''$	$\Diamond_R \varphi \to \Box_R \Diamond_R \varphi$	(5)

➼ If you look at applications of these concepts, it is some combination of these conditions that determines the properties of a particular modal base and distinguishes it from others. For e.g.

  ☞ Doxastic modal bases are considered to be consistent and fully introspective (both positive and negative introspection).

  ☞ Epistemic modal bases are consistent, fully introspective, and also realistic.

  ☞ Deontic modal bases are consistent, but are (usually) not considered introspective (see von Stechow 2004 for a view that dynamic/ability-based modal bases should have negative introspection) or realistic (because we cannot say that if something ought to be the case then it is the case).

3 Ordering sources

➼ Till now we have been studying *absolute* necessity or possibility.

➼ Coming back to Kratzer, Kratzer (1977) only provided us with the notions of modal bases, and modal force as entailment and compatibility.

➼ Then came about notions of *graded* and *comparative* modality which caused headaches for both modal logic and Kratzer (1977):

(15)　a.　There is a slight possibility that Hindi speakers will agree with Diti's interpretations of Hindi modals. (graded)

　　　b.　It is more likely that a semanticist will worry about modal logic than a phonologist will. (comparative).

◈ Neither modal logic nor Kratzer (1977) make use of comparison, thus we clearly need another component in our machinery to account for such cases.
(Although the notion of comparative similarity between possible worlds is central to the Stalnaker/Lewis theory of counterfactuals.)

◈ The added machinery: **ordering among worlds**!

◈ We intuitively can compare worlds: in the context of this talk, me sitting in front of my computer and giving the talk is of course a *better world* (/*higher ranked*) than a world where I get up and start dancing some Kathak randomly in the background.

◈ But how did I classify one as 'better/higher ranked' than the other?
☞ I used another conversational background that allowed me a ranking scheme.

    (16)   An **ordering source** is a set of propositions P in a *world* that induces a *preorder* (a binary relation on a set that is reflexive and transitive) on a set of worlds.

◈ So in w, assume there to be a conversational background g with a particular flavor, where $g(w)$ is a set of particular propositions adhering to that flavor.

    ↪ $\leqslant_{g(w)}$ is an *ordering* generated by $g(w)$.
    ↪ How?!

◈ Remember the modal base: $\bigcap f(w)$, derived from $f(w)$?
– an ordering source $g(w)$ operates **on** the modal base to yield the ranking of worlds.

◈ An ordering source determines a *partial order* on a modal base such that a world w' comes closer to the ideal set up by $g(w)$ than a world w'' iff w' makes more ideal propositions true than w'' does.

◈ **Insight:** $g(w)$ takes each world in $\bigcap f(w)$ and sees how many propositions of $g(w)$ are true in that world. The more are true, the higher the world is ranked.

◈ Let's assume a *deontic ordering source*: i.e. $g(w)$ is a set of laws $\{p, q, r\}$ that apply in w.
– $\leqslant_{g(w)}$ will render a ranking of worlds based on how many laws are being followed in each world.

◈ Disclaimer (cf. Portner 2009's fn. 9, page 64): there are two ways to read $\leqslant_{g(w)}$ –
either like natural numbers where the "better" one is first, on the left
or, parse the symbol actually as "less than or equal to" and have the "better" one second, on the right.

◈ Diti is guilty of having done both:

    (17) $\forall v, v' \in \bigcap f(w) : v \leqslant_{g(w)} v'$ iff $\{p : p \in g(w) \wedge v \in p\} \supseteq \{p : p \in g(w) \wedge v' \in p\}$
                                       ((Mihoc, Bhadra, and Falaus, 2019): (16))

      – v is ranked higher than v'

(18) $\forall w', w'' \in B_K : w' \leqslant_{g(w)} w''$ iff $\{p : p \in g(w) \land w' \in p\} \subseteq \{p : p \in g(w) \land w'' \in p\}$

<div align="right">(Bhadra 2016: (15c))</div>

  – w'' is ranked higher than w'

❖ Whichever way you choose, what we get out of this computation is a ranking of worlds that were previously sitting unranked inside the modal base.

❖ The quantificational domain of our modals now are sensitive to this ranking, and there is **another operator** that will pick the *best* worlds from this ranked set.

❖ Once the worlds in $\bigcap f(w)$ are thus ordered by *g(w)*, we can define BEST ((Mihoc, Bhadra, and Falaus, 2019), cf. Portner 2009):

(19)   **BEST**$(\bigcap f(w), g(w)) = \{v \in \bigcap f(w) : \neg\exists v' \in \bigcap f(w) : v' <_{g(w)} v\}$

❖ Thus, we have now ended up with the set of worlds that are "best"/"highest ranked" according to $\leqslant_{g(w)}$.

❖ One quick aside before we move on to the final denotation for modals in the Kratzerian framework: what happens if the ordering source is infinite?

❖ For example, let's say there is a politician who loves posing with peacocks. The more the number of peacocks in the picture the better. Then *g(w)* will look like this:

(20)   *g(w)* =

  a. p_1 = "I have at least one peacock in the picture."

  b. p_2 = "I have at least two peacocks in the picture."

  c. p_3 = "I have at least three peacocks in the picture."

  d.   .

  e.   .

❖ The result will be that there cannot be any "best" worlds in such a case, because there will always be a better world for every world!

❖ One way people have tackled the possibility of such a crash is to adopt the LIMIT AS-SUMPTION.

  ↪ It's the assumption that orderings used by *natural language* always have a "best" set.

  ↪ Heated debate about the LIMIT ASSUMPTION in the domain of counterfactuals, between Lewis (1973) in the for-it camp and Stalnaker (1987) in the against-it camp (see also Pollock 2012, Herzberger 1979, Warmbrod 1982).

  ↪ Kratzerian modalists adopt the LIMIT ASSUMPTION for practical reasons of parallelisms with other kinds of quantification and simplicity.

↪ However, we should be open to the notion since we can have bouletic interpretations like (20) possible.

➜ NOW, the moment of truth is here! Keeping all of the above in mind, **what is a Kratzerian semantics** for a necessity and possibility modal?

➜ Let's think – what components need to be represented in the denotation of *must* for instance?
– a modal base (a function from worlds to sets of propositions
– an ordering source (sometimes!; a function from worlds to sets of propositions
– a function BEST that will give us the top-ranked worlds. (i.e. "doubly relative" modality)

➜ We (Mihoc, Bhadra, and Falaus, 2019) formalized it like this; there are other conceptually similar, equivalent ways one could write it (with the Kratzerian entailment and compatibility notions, for eg.):

(21) $[\![must]\!]^w$
$= \lambda f_{\langle s,\langle\langle s,t\rangle t\rangle\rangle} . \lambda g_{\langle s,\langle\langle s,t\rangle t\rangle\rangle} . \lambda p_{\langle s,t\rangle} . \forall w' \in Best(\bigcap f(w), g(w))[p(w')]$

– *padhega, hoga, chahiye*, covert $\square$, etc would all look like this.

(22) $[\![might]\!]^w$
$= \lambda f_{\langle s,\langle\langle s,t\rangle t\rangle\rangle} . \lambda g_{\langle s,\langle\langle s,t\rangle t\rangle\rangle} . \lambda p_{\langle s,t\rangle} . \exists w' \in Best(\bigcap f(w), g(w))[p(w')]$

– *sakna, shayad, bodhoy, hoyto*, etc, would all look like this.

➜ A crucial prediction this theory makes is that the modal base and the ordering source might not match in flavor.
– is that borne out empirically?
– Yes, at least for some modals.

(23)   Portner (2009) gives these examples for *must* (not an exhaustive list):
   a.   The book must have been checked out.
        ($\bigcap f(w)$ = epistemic; *g(w)* = doxastic)
   b.   You must turn at the next light.
        ($\bigcap f(w)$ = circumstantial; *g(w)* = teleological)
   c.   I must have that painting.
        ($\bigcap f(w)$ = circumstantial; *g(w)* = bouletic)
   d.   We all must die.
        ($\bigcap f(w)$ = circumstantial; *g(w)* = empty)

➜ If the ordering source is empty, then all of the worlds compatible with the modal base are equivalent with respect to the ordering source.

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