

Modality: Patterns, Theories, Logic I

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Material partially based on [Portner \(2009\)](#), Chapters 1 & 2.

1 Introduction

- ❖ *Modality* allows us the capacity to talk about objects and events that are displaced in space and time, as well as in actuality and potentiality.
- ❖ Given one of the design features of language is *displacement* ([Hockett 1960](#)), modal expressions are all pervasive in grammars.
- ❖ Modal auxiliaries and verbs: *can, could, should, must, would, need to, have to*
Modal adverbs: *possibly, necessarily, maybe, probably, certainly*
Modal adjectives, nouns: *possible, necessary, probable, certain, need, necessity, possibility*
Propositional attitude verbs: *think, believe, hope, know, pleased*
Generics, habituals, individual-level predicates: *A linguist studies languages, Diti teaches semantics, Ayesha is hilarious*
Conditionals: *If Utpal writes a book on NPIs, then...*
Tense and aspect: *I will go to the store, We are listening to a talk, We have taken a course on modality*
Evidentiality: *It seems like there's a tiger in that shed*
Covert modality with infinitives: *Tim knows how to solve the problem* ([Bhatt 2008](#): “Tim knows how he *can* solve the problem.”)
- ❖ Let's look at some examples from Hindi:

- (1) a. *Abhi woh ghar pe hoga.* *hoga*_{epistemic}
'Now he must be home.'
- b. *Abhi usse ghar pe hona chahiye.* *chahiye*_{deontic/epistemic}
'He should be home now.'
- c. *Woh 10 minut mein 2 km daur sakta hain.* *sakta*_{ability/deontic}
'He is able to/he may run 2kms in 10 minutes.'

- d. *Mujhe 10 minut mein 2 km daurna hai.*
 V-INF *hain*_{(covert)deontic/bouletic/teleological}
 'I am required to/want to/have to run 2kms in 10 minutes.'
- e. *Tum kehte toh woh kar leta.*
 V-*ta*_{epistemic/counterfactual}
 'If you would have said, he would have done it.'
- f. *Tumhe yeh kaam karna padhega.*
 V-INF *padhega*_{deontic/teleological}
 'You have to/need to do this work.'

(2) *Ram yeh kar sakta hain.*

- Ram can do it, i.e. he is physically able to.
- Ram can do it, i.e. now he is allowed to/I am giving him permission to.
- Ram may/might do it, i.e. I am not sure if he certainly will but there is a possibility given what I know about him.
- Ram may do it, i.e. the moral ethics we live by do not prevent him from doing this.
- Ram can do this to achieve a particular goal.

❖ What we are already noticing is massive amounts of ambiguity/underspecification in *modal flavor*.

❖ Then should we consider the existence of these patterns as a result of accidental polysemy?

❖ Kratzer (1981, 1991): No! This is the result of **contextual dependency**.

☞ Modals by themselves have a skeletal meaning denotation; together with essential components from the context, a modal gets the particular flavor that it has in that particular context.

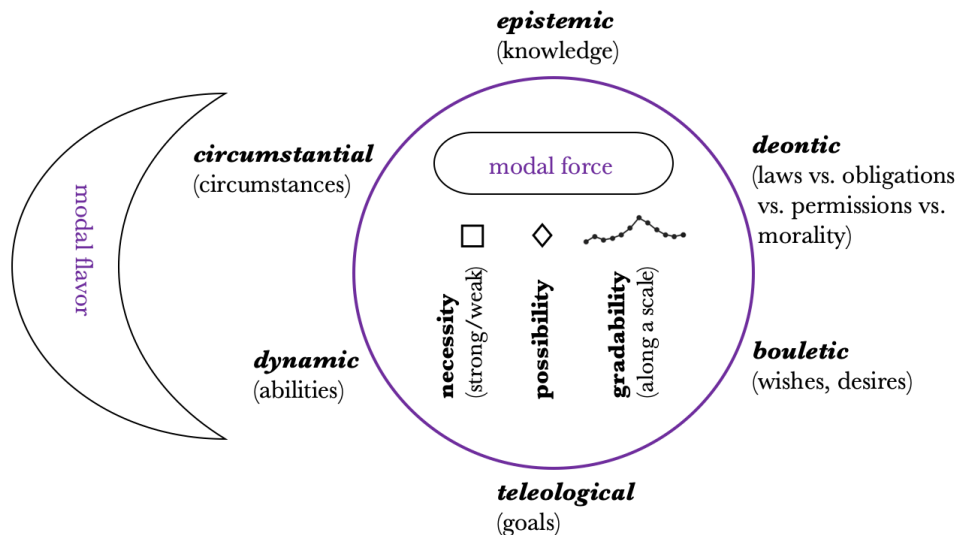


Figure 1: Modal *force* vs. modal *flavor*

- ❖ Any theory of modality in natural language has to explain how to account for these different flavors within the same framework, and correlate them with differences in modal force.
- ❖ We are going to look at two influential schools of thought bearing on this issue: modal logic (accessibility relations in particular), and natural language semantics (possible worlds in particular).
- ❖ There are many many aspects and facets of modality we will not be able to cover in this workshop: their interaction with scopal operators like *negation, quantifiers, conditionals, questions*, etc, their direct interaction with tense and aspect, their interaction with disjunction (*free choice*), clause-type related phenomena like *modal subordination*, relationship with *imperatives*, the very related phenomena of *evidentiality*, etc.

2 Accessibility Relations

- ❖ *Logic* comprises studying systems of reasoning; *Modal Logic* focuses on reasoning involving the concepts of necessity, possibility, implication, obligation, etc. (Portner 2009).
- ❖ Modal logic is not the same thing as the linguistics of modal expressions!
 - ❖ Modal logic aims to capture patterns of reasoning irrespective of natural language words like *must, possible, sakna, hona, zaroori, dorkar, uchit, uchitam*, etc.
 - ❖ Once you start studying the formal properties of operators like \Box and \Diamond , it can take you far far beyond the semantics of human language.
- ❖ In a modal logic language (MLL; Portner 2009), we first begin with some essential tools:
 - (3) Included in such a MLL are:
 - a. Infinite number of propositional variables: $p, q, r, s, t \dots$
 - b. Negation: if α is a sentence in MLL, then so is $\neg \alpha$.
 - c. Conjunction, disjunction, conditionals: if α and β are sentences of MLL, then so are $\alpha \wedge \beta, \alpha \vee \beta, \alpha \supset \beta$.
 - d. Necessity and Possibility: if α is a sentence of MLL, then so are $\Box \alpha$ and $\Diamond \alpha$.
- ❖ Note that some of these are *derivable* from other, more basic operators:
 - (4)
 - a. $\alpha \wedge \beta = \neg (\alpha \vee \neg \beta)$
 - b. $\alpha \supset \beta = \neg (\alpha \wedge \neg \beta)$
 - c. $\Box \beta = \neg \Diamond \neg \beta$
- ❖ Next, we come to two concepts that will get the “modal” part of things rolling (Hughes and Cresswell 1996):

(5) FRAMES

A frame F is a pair $\langle W, R \rangle$ where W is (usually) a set of possible worlds, and R is a relation on W .

❖ This relation R is ultra important because it provides the mechanism for the worlds to “talk” to each other. For example:

- (6) a. **Reflexive frame:** $\langle W, R \rangle$ is a reflexive frame iff for every $w \in W$, $R(w, w)$.
b. **Symmetrical frame:** $\langle W, R \rangle$ is a symmetrical frame iff for every w and $u \in W$, if $R(w, u)$, then $R(u, w)$.
c. **Serial frame:** $\langle W, R \rangle$ is a serial frame iff for every $w \in W$, there is a $u \in W$ such that $R(w, u)$.
d. **Transitive frame:** $\langle W, R \rangle$ is a transitive frame iff for every $w, u, v \in W$, if $R(w, u)$, and $R(u, v)$, then $R(w, v)$.
e. **Equivalence frame:** $\langle W, R \rangle$ is an equivalence frame iff it is a reflexive, symmetrical, and transitive frame.

❖ Think of these frames as ways a world can talk to/access its friends that are far away from it.

❖ Different types of R thus gives us different types of **accessibility relations** (cf. Kripke semantics – a system created by Saul Kripke and Andre Joyal in late 1950s).

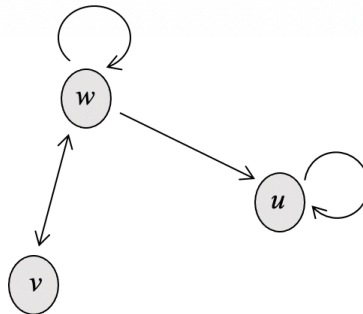


Figure 2: Accessibility between worlds

❖ Can we figure out the different relations/frames that exist between the worlds w, u, v ?

❖ With the concept of a FRAME in place, we can define a MODEL:

(7) MODEL

A model M is a pair $\langle F, V \rangle$, where F is a frame and V is a valuation function that takes a proposition in a $w \in W$ and returns a value of 1 or 0.

- ❖ For example, $V(w, p) = 1$, $V(u, q) = 0$, etc.
- ❖ We can do this for all atomic propositions as well all derived complex propositions containing operators like $\wedge, \neg, \tilde{N}, \Box, \Diamond$.
- ❖ We also have the useful logical property of **validity** in modal logic:

- (8) a. A sentence α is VALID in a MODEL $M = \langle W, R, \nu \rangle$ iff $\forall w \in W, V(\alpha, w, M) = 1$.
- b. A sentence α is VALID on a FRAME F iff, for every valuation function V , α is valid in the model $M = \langle F, V \rangle$.

- ❖ Thus, validity in a model is separate from validity on a frame (changing the valuation function changes the latter, for instance).¹
- ❖ Take a reflexive frame, and if a sentence is valid on such a frame then it is T-VALID:

- (9) A T-VALID sentence: $\Box p \tilde{N} p$
 Natural language counterparts:
- a. If Ram must be at home now, then he is at home now.
 (where *must_{epistemic}*)
- b. If Sita must eat 10 rasgullas to win, then she will eat 10 rasgullas.
 (where *must_{deontic}*)

- ❖ Only (9-a) is T-VALID because the sentence is true, while (9-b) is not T-VALID because the sentence is not true.
 - ❖ This tells us that T-validity can be useful for epistemic logic, but not for deontic logic.

- ❖ Take a serial frame, and if a sentence is valid on such a frame then it is D-VALID:

- (10) A T-VALID and D-VALID sentence: $\Box p \tilde{N} \Diamond p$
 Natural language counterparts:
- a. If Ram must be at home now, then he might be at home now.
 (where *must_{epistemic}*, *might_{epistemic}*)
- b. If Sita must eat 10 rasgullas to win, then she may eat 10 rasgullas.
 (where *must_{deontic-obligation}*, *may_{deontic-permission}*)

- ❖ With the flavors and subflavors as indicated, the sentence with deontic modals in (10-b) being D-VALID suggests that D-validity can be a property of deontic logic.

¹There can be different kinds of validity based on the specific types of frame: K-VALID, T-VALID, B-VALID, D-VALID, S4-VALID, S5-VALID (Lewis 1918, Gödel 1933, Blackburn et al. 2002).

- ⊕ Note that the sentence with the epistemic modals are thus both T-valid and D-valid (more on this on page 6 below).
- ⊕ Our main stars for today, \Box and \Diamond , impose a **layer of quantification** on the accessibility relations between worlds:
 - (11) $\forall \Box p w^{w,M} = 1$ iff for **every world** in W accessible to w , p holds (/is true) in all of those worlds.
 - ⊕ Crucially note that we have not said *what type* of accessibility relation holds between the worlds – this could be any type!
 - ⊕ So what is the \Box giving us – just the *quantity* of accessible worlds where the proposition in its scope holds.
This is thus a natural way to think about **modal force**.
 - (12) $\forall \Diamond p w^{w,M} = 1$ iff for **some world** in W accessible to w , p holds (/is true) in that world.
- ⊕ Now it's time to wonder: where does **modal flavor** then come from in this framework of modal logic?
 - ☞ From the refined definitions of the accessibility relation R in terms of knowledge, rules, permissions, obligations, goals, desires, circumstances, abilities, etc.
- ⊕ Thus, our frames and accessibility relations can now have special designations: (Note: I am using Portner's style of descriptions below (cf. [Portner 2009](#)) but most styles are sons and daughters of the formalizations in Kripke semantics; cf. also [Hughes and Cresswell 1996](#), [Blackburn et al. 2002](#)).
- (13) **Epistemic frame**
 $F = \langle W, R \rangle$ is an epistemic frame iff for some individual i :
 - a. W = the set of possible worlds conceivable by humans.²
 - b. R = the relation which holds between two worlds w and u iff everything which i knows in w is also true in u .
- ⊕ This R inside this EPISTEMIC FRAME can be called an EPISTEMIC ACCESSIBILITY RELATION.
- ⊕ What about the different properties of frames we learnt in (6)?
 - ☞ They can be applied to each of these relations.
- ⊕ For example, *knowledge* is considered to have the property of reflexivity, because if an agent *knows* a proposition in w , then it is true/is a fact in w .
– thus, we can say that the epistemic frame is reflexive.

²Leibniz' work on modal metaphysics takes a possible world to a complete way that the universe could be through its history; possible worlds accessible from our world could have minimal details changed (like maybe today is a Tuesday) or be completely different (maybe dinosaurs are still alive). Linguists ignore inconceivable worlds.

- ❖ Actually, if we assume an idealized account of knowledge where agents are perfect reasoners with infinite memory, then we can say that the epistemic frame actually contains an *equivalence* relation.
 - (for example, recall that sentences with epistemic modals were both T-VALID and D-VALID to start with. cf. (10))
 - but we know that that knowledge derived from different types of evidence and inferences sometimes admit errors, etc. (von Fintel and Gillies, 2010; Mihoc, Bhadra, and Falaus, 2019)
- ❖ With similar tools, we can define a DEONTIC FRAME and a corresponding DEONTIC ACCESSIBILITY RELATION inside it (assuming we are in the realm of rules for instance):

(14) **Deontic frame**

$F = \langle W, R \rangle$ is a deontic frame iff for some system of rules r :

- a. W = the set of possible worlds conceivable by humans.
- b. R = the relation which holds between two worlds w and u iff all of the rules which are established by r (the relation that associates each world in W with a set of rules) in w are followed in u .

- ❖ Again, thinking about properties of this frame, deontic frames are *serial*:
 - by invoking seriality we obliterate the possibility of having an inconsistent set of rules
 - because for every rule, there is a world in which the rule is followed (because the property of seriality requires that there be a corresponding world for every world accessible by the deontic accessibility relation).
- ❖ A disclaimer again: this is an idealized notion of a deontic relation, because we might find ourselves with contradictory requirements in reality.
- ❖ Now, with these tools in place, how do we apply them to our modal expressions?

- (15) a. Necessity modals (\square): *must, should, would, zaroori, V-INF chahiye, V-INF padhega, nischoi, dorkar, uchit, uchitam, etc.*
- b. Possibility modals (\diamond): *may, might, can, could, sakna, shayad, hoyto, bodhoy, etc.*

- ❖ The central insight: for each modal flavor and *subtypes* of each flavor, there has to be a separate accessibility relation.

- ❖ Formally:

(16) $R_{epist}(w) = \{w' \mid w' \text{ is a world in which all the known facts in } w \text{ hold}\}$

(17) $R_{deontic}(w) = \{w' \mid w' \text{ is a world in which all the rules in } w \text{ are followed}\}$

- ❖ Within deontic necessity, there is a distinction between **strong** and **weak** necessity (*must, necessary, have to, etc.* vs. *should, ought*).

- ❖ Test 1 for detecting strength: strong necessity modals can reinforce weak ones but not vice-versa (von Fintel and Iatridou 2008):

- (18) a. You should wash your hands, in fact you must.
 b. ??You must wash your hands, in fact you should.

- ❖ Test 2 for detecting strength: Weak necessity modals are compatible with the negation of strong ones, but not vice versa (von Fintel and Iatridou 2008):

- (19) a. You ought to/should do the dishes, but you don't need to/have to.
 b. ??You need to/have to do the dishes, but it's not the case that you should/ought to.

- ❖ For an analysis of differential strength in deontic necessity in Bangla (with connections to Telugu and Hindi), see Bhadra and Banerjee (2020).

- ❖ Within modal logic, can we capture this relationship?: $must\ p \mid\! \cup\ should\ p$

- ❖ Let r_{must} be the set of rules on which the accessibility relation of *must* – $R_{must}(w)$ is based on; and let r_{should} be the set of rules on which the accessibility relation of *should* – $R_{should}(w)$ is based:

$$(20) \quad R_{must}(w) = \{w' \mid @w' s.t. Rpw, w'q, Vppqpw'q \quad 1u \quad \text{“} \quad r_{must}$$

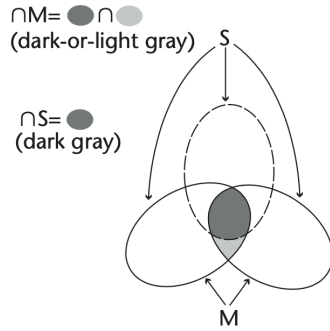
$$(21) \quad R_{should}(w) = \{w' \mid @w' s.t. Rpw, w'q, Vppqpw'q \quad 1u \quad \text{“} \quad r_{should}$$

- ❖ Both are deontic necessity modals (\square), thus crucially there is no difference in the denotations of the modals;
 - the main action (/difference) is in the sets of rules behind the accessibility relations. (Keep this insight in mind – it will carry over to the other school of thought we will be exploring tomorrow).

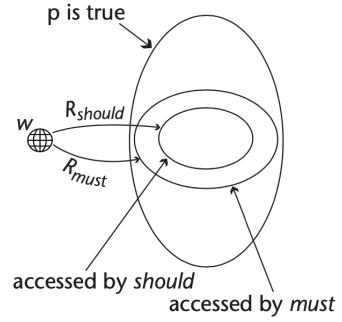
- ❖ Steps (let's name r_{must} as M and r_{should} as S):

- (22) a. if M ,, S
 b. then $R_{should}(w)$,, $R_{must}(w)$
 c. i.e. S ,, M
 d. i.e. the subset relation reverses once we take intersections

- ❖ Now look at Portner (2009)'s representations of the relationship:



Intersection reverses the subset relation



Deontic *must p* entails deontic *should p*

- ❖ In the first figure: M contains less rules thus only the two solid ovals, while S contains more rules thus all three ovals.
 - ❖ Thus, “ M is larger than the “ S , confirming the subset relationship in (22-c).
 - ❖ **Insight:** the more things you care about, the smaller the intersection of worlds will be where all of those things are true.
 - ❖ Thus, we can confirm $\text{must } p \mid \cup \text{should } p$.
 - provided that there are at least two different kinds of deontic accessibility relations
 - figure on the right
 - ❖ But deontic modality can have many other subtypes, and thus even these two deontic accessibility relations are not enough!
 - apart from the two kinds within $R_{\text{deontic-rules}}$, there has to be $R_{\text{deontic-permissions}}$, $R_{\text{deontic-obligations}}$, $R_{\text{deontic-morals}}$, etc.
- (23) Just splitting apart deontic *must*:
- a. [In view of the laws of Minnesota], drivers must yield to pedestrians.
 - b. [In view of the traditions of our family], you, as the youngest child, must touch the feet of all elders.
 - c. [In view of the rules of student-teacher relationships], you must not yell at your teachers.

- ❖ Kratzer (1977) points out that the kind of restrictions in []-s that determine differences in accessibility relations can be infinitely many.
- ❖ **So what is starting to look like a problem here?**
 - ☞ Massive over-generation!
- ❖ Additionally, not to forget that there are several other flavors outside of deontic modality!:

(24) a. Dogs **can** swim. (ability/dynamic)


- b. Given how much you love semantics, you **should** attend the modality workshop. (desire)
- c. To get into JNU, you **have to** study hard. (goal)
- d. A pandemic **may** eventually wipe out large sections of humanity. (history)

(25) $R_{bouletic}(w) = \{w' \mid w' \text{ is a world in which all the desires of an agent } i \text{ in } w \text{ are satisfied}\}$

- ❖ The massive ambiguity/underspecification problem that we started out with is exacerbated by the possibility of the generation of multiple accessibility relations for the same modal:

(26) *Ram yeh kar sakta hain.*

- a. $R(w, v)$, where $R_{dynamic}$ $\{w_1, w_2, w_3, w_4, \dots\}$
- b. $R(w, v)$, where $R_{deontic-permissions}$ $\{w_5, w_6, w_7, w_8, \dots\}$
- c. $R(w, v)$, where $R_{epistemic}$ $\{w_9, w_{10}, w_{11}, w_{12}, \dots\}$
- d. $R(w, v)$, where $R_{deontic-morality}$ $\{w_{13}, w_{14}, w_{15}, w_{16}, \dots\}$
- e. $R(w, v)$, where $R_{teleological}$ $\{w_{17}, w_{18}, w_{19}, w_{20}, \dots\}$

- ❖ And then within modal logic, what prevents 10 other accessibility relations from being generated for *sakna*?³
- ❖ Thus, we understand the nature of the formal relations and properties better but have we made real progress towards providing an account of **modality in natural language**?
- ❖  Unfortunately, no! Because our current logical system predicts many many more meanings for modals than they actually have.
- ❖ Enter: the field-changing work of Angelika Kratzer.
(tomorrow's journey)

³We have not even discussed the *indexicality* of modals (relativization to contextual parameters like speakers, hearers, times, locations); imagine how much more over-generation there can be once these factors are wired into the semantics.

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